BALKANS CAPITAL MARKETS AND MARKET RISK FORECASTING UNDER LONG MEMORY IN RETURNS

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ABSTRACT

The Basel Committee on banking supervision at the Bank for International Settlements requires financial institutions to meet capital requirements on base VaR estimates, which has made the VaR methodology a fundamental market risk management tool employed by the financial institutions.

Although it is widely used, the practicability of VaR was questioned and the traditional approaches to VAR computations – the historical simulation, variance-covariance method, Monte Carlo simulation and stress-testing – were claimed to provide a non-satisfactory evaluation of possible losses for stock markets with long memory in returns.

This paper presents an empirical analysis of the value-at-risk in the financial environment of the regulated financial markets on the Balkans (Turkey, Croatia, Romania and Bulgaria). The results obtained for the considered stock exchange indices BET, CROBEX, ISE100 and SOFIX indicate presence of long-term dependencies in the logarithmic returns and variance, which means the returns are featured by the so called "fat tails" and respectively the assumption of a normal distribution of returns is inappropriate. At all indices under survey the $VaR_{(1\%)}$ *estimates which were calculated for the period* Q_1*2002-*Q_1*2014 through historical simulation and under the assumption of normal and Student t distribution of the returns underestimate the actual market risk and respectively the results we have received evaluate the models as inaccurate. When estimating a Monte Carlo simulation which includes a model of the conditional heteroscedasticity without any long-term dependency the tests for adequacy of the model do not give unambiguous results. The models of conditional heteroscedasticity proposed by the author consider the long-term dependency computed it in three different classical methods (Rescaled-Range Analysis (Hurst Method), Whittle Method and Wavelets Method). They pass successfully through the test of adequacy and generally provide more accurate* $VaR_{(1\%)}$ *forecasts.*

For the purposes of this analysis the following statistical tests are used: Kupiec's test-likelihood ratio unconditional coverage test and Christoffersen's test - likelihood ratio independence coverage and likelihood ratio conditional coverage test.

Keywords: Value at risk, Return distributions, Long memory

JEL Classification: C53, G15.

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INTRODUCTION

Studies of market risk in the financial environment of regulated stock markets are particularly relevant against the backdrop of the global financial crisis of 2007 and the subsequent sovereign debt crises in Greece (since 2010), Ireland and Portugal (since 2011) and Spain (from 2012). This is even more effective for Bulgaria and the regulated capital markets of the Balkans (Turkey, Croatia and Romania) in view of the fact that most of them are small and newly arisen regional markets. An exception to this is the stock market in Turkey, which is an established regional market and for it can be expected much weaker effects caused by its neighboring regional markets.

Patev et al. (Plamen Patev, Nigokhos Kanaryan and Katerina Lyroudi, 2009) have examined the Bulgarian stock market risk over the period 24 October 2000 – 19 November 2004. The result of their research shows that the SOFIX index has basic characteristics that are observed in most of the emerging stock markets, namely: high risk, significant autocorrelation, non-normality and volatility clustering. Three models have been applied to assess and estimate the Bulgarian stock market risk: RiskMetrics, EWMA with t-distributed innovations and EWMA with GED distributed innovations. The results revealed that the EWMA with t-distributed innovations and the EWMA with GED distributed innovations evaluate the risk of the Bulgarian stock market adequately.

Zivkovitch et al. (Zivkovitch, Measuring market risk in EU new member states, 2007) applied VaR methodology and historical simulation on the Croatian stock market indices in an effort to measure Value-at-Risk. Zivkovitch et al. has also analyzed VaR models for ten small and newly arisen regional markets and concluded that use of common VaR models to forecast VaR is not suitable for transition economies. (Zivkovitch, Testing popular VaR models in EU new member and candidate states, 2007).

Kasman (Kasman, 2009) has examined long memory property of the Turkish futures market and the estimation results provide evidence supporting the FIGARCH models, in the sense that the FIGARCH models fit the data series better than the GARCH models. The results of the FIGARCH model show that estimates of the long memory parameters are significantly different from zero, suggesting that volatility series are long memory processes in the Turkish futures market. The estimation results also indicate that the skewed Student-t distribution outperforms the normal distribution. The VaR values have also been estimated using the FIGARCH (1, d, 1) model with three distributions. Comparing the estimated in-sample and out-of-sample VaR values based on Kupiec's LR test, the skewed Student-t model performs better than the normal distribution in describing the return series in the Turkish futures market. In summary, since long memory model outperforms the traditional short-memory model risk analyzing methods requiring variance series, such as VaR, provide more efficient results when the variance series of the ISE-30 index futures returns is filtered by the long memory model, rather than by the short memory model. Therefore, these findings would be helpful to the financial managers, investors and regulators dealing with the Turkish futures market.

Dorich (Dorich, 2011) has examined several alternative models of return distribution for BELEX15 to compare the predictive ability of the VaR estimates based on them. In the case of BELEX15 index returns asymmetric behavior was not discovered. Since the distribution of the log-returns exhibits leptokurtosis, several models of leptokurtic distribution were chosen: Student t, NIG, hyperbolic and stable. For both tails NIG distribution is the closest one to the empirical data. Based on Dorich results the Student t and NIG distributions are acceptable for all considered α - values. Although static models cannot reproduce volatility clustering, they may be successful in modelling tails of distribution and computing VaR of the Belgrade Stock Exchange index BELEX15.

Such studies are very useful for investors who are trying to compile portfolios of global assets with the requirement to be resilient in times of crisis. An interesting question is whether in such times of crisis, the market risk of the stock markets in the Balkans region differs significantly from that of the developed world's stock markets.

METHODOLOGY

The sense in which we use the "market risk" concept is that of a specifically selected measure of risk which is numerically measurable and for each of the examined stock markets (represented by their major stock indexes) we can collate a number that we call risk, determined for a portfolio of the respective stock market indexes.

In this context, let us consider a discrete random time series X_1, X_2, $X_{3 \ldots}$ … $X_{t \ldots}$ …, X_{t+m} which consists of closing prices of our sample indices and its logarithmic return at time *t* is:

$$R_t = \log \frac{X_t}{X_{t-1}}, \quad t = 2,3,\ldots,n \tag{1}$$

Historically, the oldest numerically quantifiable measure of market risk, proposed by Markowitz (1952) in the context of his classic works for portfolio optimization, is the standard deviation (variance) of a random variable. The dispersion of a random variable is a measure of the distribution of a random variable, i.e. its deviation from the mathematical expectation.

The Basel Committee on banking supervision at the Bank for International Settlements requires financial institutions to meet capital requirements on base VaR estimates, which has made the VaR methodology a fundamental market risk management tool employed by the financial institutions. According to Basel II framework, the preferred approach for market risk is value-at-risk (VaR). Banks will have the flexibility in devising the precise nature of their models, but the following minimum standards will apply for the purpose of calculating their capital charge: "Value-at-risk" must be computed on a daily basis, a 99th percentile, one-tailed confidence interval is to be used. An instantaneous price shock equivalent to a 10 day movement in prices is to be used, the historical observation period is a minimum length of one year and the banks should update their data sets no less frequently than once every three months. In particular VaR is defined for a fixed confidence level α ∈ (0, 1) as the smallest number $\mathcal{V}$, such that the probability that the loss L exceeds $\mathcal{V}$ is not larger than (1-α). If a random variable X describes a random return, then VaR is defined as the negative value of the lower α-quantile of the distribution of returns.

$$VaR_\alpha(X) = -\inf_x \{x \mid P(X \leq x) \geq \alpha\} = -F_X^{-1}(\alpha)$$

where $\alpha \in (0,1)$ and $F_X^{-1}(\alpha)$ is the inverse function of the distribution of the random variable X.

Despite the wide variety of approaches to calculating VaR estimates, we could consider them in four main groups. They differ among themselves in the assumptions made regarding the statistical properties of the time series formed by the empirical data and also in the approaches to construct a distribution function $F_X(\alpha)$. These groups are: the non-parametric historical approach, parametric approaches which could be analytical or could be based on a model, Monte Carlo stochastic simulations and stress tests based on scenarios. In this study the author has confined himself to the first three approaches and the markets under survey are not analyzed through stress tests.

THE NON-PARAMETRIC HISTORICAL APPROACH (HISTORICAL SIMULATION)

The simplest from computational viewpoint is the non-parametric historical approach that does not assume any specific distribution of the returns of the financial assets which we consider and the VaR forecasts are determined based only on the historical returns. When the historical simulation (HS) approach is applied the distribution of the logarithmic returns R_t is estimated within the settings of the distribution of the empirical historical data $\mathbf{X}_{t-n+1},...,\mathbf{X}_t$. Thus, the method does not rely on any parameterized assumptions about the distribution of the returns. However this does not mean that stationarity is not implied for the discrete stochastic series of the empirical historical data $\mathbf{X}_{t-n+1},...,\mathbf{X}_t$, which is a necessary condition to ensure convergence of the distribution of the empirical returns and of the distribution of the real returns. When we apply this approach we order the possible realizations of the logarithmic returns $R_1<R_2< R_3$ … … … $<R_n$ as an increasing statistical series and we set an appropriate level of confidence α. The value of the quantile corresponding to the chosen level of confidence is actually the VaR of the asset. For example, in a simulation for a period of 1000 work days, if we want to calculate the VaR at 99% confidence level, the VaR value is simply the 10^{-th} lowest returns value.

PARAMETRIC APPROACHES

The simplest of the computational point of view and at the same time the most widespread parametric approach is that of constructing a distribution function $F_X(\alpha)$ with the assumption for normal distribution of the returns. Then the VaR forecasts are completely determined by two parameters - the mathematical expectation μ and the standard deviation σ. Thus to construct the distribution function we are using the following formula:

$$F_X(\alpha) = \frac{1}{\sqrt{2\pi\sigma}} exp\left\{-\frac{(\alpha-\mu)^2}{2\sigma^2}\right\} \tag{2}$$

$$\hat{\mu} = \frac{1}{T}\sum_{n=1}^{T} R_n, \quad \hat{\sigma} = \left(\frac{1}{T-1}\sum_{n=1}^{T}(R_n-\hat{\mu})^2\right)^{1/2} \tag{3}$$

Despite its simplicity and wide spread, the assumption for a normal distribution is problematic, since the returns on most assets have distributions with strong skewness and kurtosis (fat tails). If we have such a distribution with fat tails, the VaR estimates obtained by this method will underestimate the maximum possible loss. This method uses the historical standard deviation, which makes it unsuitable in times of crisis and more generally in times of dynamic changes in the market conditions.

One possible approach for modeling of empirical distributions with strong skewness and kurtosis (fat tails) is the one in which in order to construct a distribution function $F_X(\alpha)$ we have to assume Student *t* distribution of the returns. Under this assumption we can construct a distribution function using the formula:

$$F_X(\alpha) = \frac{\Gamma(\frac{\nu}{2}+\frac{1}{2})}{\Gamma(\frac{\nu}{2}\sqrt{\pi\nu\beta})} . \left(1+\frac{(\alpha-\mu)^2}{\nu\beta}\right)^{-\frac{(1+\upsilon)}{2}} \tag{4}$$

ECONOMETRIC APPROACHES

The presence of statistically significant conditional heteroscedasticity in the empirical data for the regulated capital markets we have reviewed allows us to use econometric ARMA-GARCH approaches to construct the distribution function $F_X(\alpha)$. To forecast ε_t we could use an asymmetric ARMA-GARCH model and the conditional dependency of the error in the moment t on the previous realizations $e_{t-1}, e_{t-2}, \ldots$. When we use an ARMA model for the deterministic part of X_t and denote the forecast for the variance $E_{t-1}(e_t^2)$ with σ_t^2 then we get the following system of equations:

$$X_t = C_1 + \sum_{i=1}^{p} \phi_i X_{t-i} + e_t + \sum_{j=1}^{q} \theta_j e_{t-j} \tag{5}$$

$$\sigma_t^2 = C_2 + \sum_{i=1}^{P} G_i \sigma_{t-i}^2 + \sum_{j=1}^{Q} A_j e_{t-j}^2 + \sum_{j=1}^{Q} L_j S_{j-t} e_{t-j}^2 \tag{6}$$

and S_j= 1 if $e_{t-j} < 0$, S_j = 0 if $e_{t-j} \geq 0$, and the following conditions hold:

$$\sum_{i-1}^{P} G_i + \sum_{j-1}^{Q} A_j + \frac{1}{2}\sum_{j-1}^{Q} L_j < 1, C_2 \geq 0, G_i \geq 0, A_i \geq 0, A_i + L_j \geq 0 \tag{7}$$

where e_t is the random disturbance or the error for a one-step ahead forecast, X_{t-1}, X_{t-2}, … are the past values of the time series, and e_{t-1}, e_{t-2}, … are previous realizations of the error.

The first of the above equations is an ARMA (p, q) model, where ϕ_i stay for the autoregressive coefficients and Θ_j stay for the coefficients of type "moving average".

The second equation models the behavior of the conditional variance and A_j are employed to denote the coefficients of the ARCH part while G_i - the additional coefficients that are introduced by Bollerslev - to represent the influence of the past forecasts $\sigma_{t-1}, \sigma_{t-2}, \ldots, \sigma_{t-m}$.

The distribution of the residuals is often found to be leptokurtic. As an "ad-hoc approach" the innovations can be modeled by a t-distribution where the degree-of-freedom parameter is estimated with maximum likelihood. This approach works quite well for return series with symmetric tails but fails when the tails are asymmetric. McNeil, A. and Frey, R (McNeil, A. and Frey, R., 2000) have proposed the generalized Pareto distribution (GPD) approximation which employs the extreme value theory to model the tail of the distribution of the innovations.

The definition of the probability density function for the GPD with shape parameter $k \neq 0$, scale parameter σ, and threshold parameter θ, is:

$$y = f(x\,|k,\sigma,\theta) = \left(\frac{1}{\sigma}\right)\left(1+k\frac{(x-\theta)}{\sigma}\right)^{-1-\frac{1}{k}} \tag{8}$$

for $\theta < x$, when $k > 0$, or for $\theta < x < -\sigma/k$, when $k < 0$

For $k = 0$, the density is

$$y = f(x\,|0, \sigma, \theta) = \left(\frac{1}{\sigma}\right) e^{-\frac{(x-\theta)}{\sigma}} \qquad \text{for} \qquad \theta < x. \tag{9}$$

MONTE CARLO SIMULATION METHOD

The Monte Carlo method simulates the behavior of risk factors and asset returns by generating random returns paths. Monte Carlo simulations provide possible index values on a given date t+n after the present time t; n>0. The VaR value can be determined from the distribution of simulated index values. The Monte Carlo approach is performed according to the following Algorithm 1:

1. Specify an asymmetric stochastic AR (1)/GJR (1, 1) process that models well the dynamics of the capital markets under investigation. Additionally, the standardized residuals of each index are modeled as a standardized Student's t distribution to compensate for the fat tails that are often associated with equity returns.
2. Having filtered the model residuals from each return series, standardize the residuals by the corresponding conditional standard deviation. These standardized residuals represent the underlying zero-mean, unit-variance, i.i.d. series upon which the Extreme Value Theory (EVT) estimation of the tails and sample cumulative distribution function (CDF) of each asset using a generalized Pareto distribution (GPD) estimate for the upper and lower tails.
3. Then, by extrapolating into the generalized Pareto tails and interpolating into the smoothed interior, transform the uniform random variables to standardized residuals via the inversion of the semi-parametric CDF of each index. This produces simulated standardized residuals consistent with those obtained from the AR (1) / GJR (1, 1) filtering process above.
4. 1000 independent random trials of dependent standardized index residuals over a one trading day horizon are simulated.
5. Using the simulated standardized residuals as the i.i.d. input noise process, reintroduce the autocorrelation and heteroscedasticity observed in the original index returns.
6. Having simulated the returns of each index, compute the VaR at 1% confidence level, over the one trading day risk horizon.
7. Repeat steps 1 to 6 many times to form the distribution of the VaR over the 1569 trading days horizon with 1024 trading days window.

VaR UNDER LONG MEMORY IN RETURNS

The presence of a short-term dependency in a given data set could be modeled very well by the classical ARIMA processes however the covariance between the observations X_i and X_{i+h} decreases fast with the increase of h. More precisely – the autocorrelation function of the process $\rho(k)$ is geometrically restricted:

$$|\rho(k)| \leq Cr^k, \quad k = 1,2,\ldots, \qquad \text{where } C>0 \text{ and } 0<r<1. \tag{10}$$

A class of models where the covariance between distant observations decreases like a power function, are suggested simultaneously by (Hosking, 1981) and (Granger,C.W.; Joyeux,R., 1980). A main feature of these models is the usage of fractional differentiation. The operator for fractional differentiation is formally defined by the following binomial decomposition:

$$\nabla^d = (1-B)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-B)^j = 1 - dB - \frac{1}{2}d(1-d)B^2 - \frac{1}{6}d(1-d)(2-d)B^3 - \ldots \tag{11}$$

where B is the lag operator $Bx_i = x_{i-1}$, and d takes fractional values. To calculate the binomial coefficients it is technically more convenient to use the Gama function $\Gamma(.)$:

$$\nabla^d = (1-B)^d = \sum_{j=0}^{\infty} \pi_j B^j, \text{ where} \tag{12}$$

$$\pi_j = \prod_{0<k\leq j} \frac{k-1-d}{k} = \frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)}, \quad j = 0,1,2,\ldots, \tag{13}$$

The ARIMA (0, d, 0) process could be defined when the operator for fractional differentiation is used (in the case of Gaussian innovations): $\nabla^d X_t = Z_t$,

where Z_t is a process of discrete white noise – for simplicity it is taken to have one as a dispersion and d takes values in the (-0.5,0.5) interval.

The main features of one ARIMA (0, d, 0) process could be listed without a detailed exposition as follows:

- when d<0.5 $\{X_t\}$ is a stationary process with infinite moving average representation:

$$X_t = \sum_{j=0}^{\infty} \psi_j Z_{t-j} = \nabla^{-d} Z_t, \text{ where} \tag{14}$$

$$\psi_j = \frac{(j-1+d)!}{j!(-1+d)!} = \frac{\Gamma(j+d)}{\Gamma(j+1)\Gamma(d)} \tag{15}$$

when $j \to \infty, \quad \psi_j \sim j^{d-1} \big/ (d-1)!$.

- when d>-0.5 $\{X_t\}$ is an invertible process and has the following infinite autoregression representation: $\nabla^d X_t = Z_t = \sum_{j=0}^{\infty} \pi_j X_{t-j}$, where the coefficients π_j are defined in (8) when $j \to \infty$, $\pi_j \sim j^{-d-1} \big/ (-d-1)!$
- when -0.5<d<0.5 the spectral density of $\{X_t\}$ is $s(\omega) = \left(2\sin\frac{1}{2}\omega\right)^{-2d}$, $0 < \omega \le \pi$, and in case of $\omega \to \infty$ we have that $s(\omega) \to \omega^{-2d}$.
- when -0.5<d<0.5 the autocovariance function, autocorrelation function and the partial autocorrelation function are:

$$\gamma(h) = E(X_t X_{t-h}) = \frac{(-1)^h (-2d)!}{(h-d)!(-h-d)!}$$

$$\rho(h) = \frac{\gamma(h)}{\gamma(0)} = \frac{\Gamma(h+d)\Gamma(1-d)}{\Gamma(h-d+1)\Gamma(d)}, \text{ and in case } h \to \infty \text{ we have that} \qquad (16)$$

$$\rho(h) \sim \frac{d!}{(d-1)!} h^{2d-1}; \ \alpha(h) = \frac{d}{(h-d)}.$$

The listed features reveal that when -0.5<d<0.5 the ARIMA (0, d, 0) process is stationary and invertible, with coefficients ψ_j, π_j that decrease like a power function with the increase of j. We should note the difference from the exponential decrease in the case of a standard ARIMA (p, 0, q) process. When d>0 there is a long-term dependency, as could be seen from the formulas for $s(\omega)$ if $\omega \to 0$ and $\rho(h)$ if $h \to \infty$.

A significantly broader class of ARIMA (p, d, q) processes with fractional d could be defined on the basis of the results received for ARIMA (0, d, 0).

The process $\{X_t\}$ is a fractional ARIMA (p, d, q) process with -0.5<d<0.5 if it is stationary and satisfies a difference equation of the form:

$$\Phi(B)\nabla^d X_t = \Theta(B) Z_t \quad \text{where} \qquad (17)$$

$$\begin{aligned} \Phi(B) &= 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p, \\ \Theta(B) &= 1 + \theta_1 B + \theta_2 B^2 + \ldots + \theta_q B^q, \end{aligned} \qquad (18)$$

B is the lag operator, and Z_t is a discrete white noise.

If the polynomials $\Phi(B), \Theta(B)$ do not have common roots then in case of $\Phi(z) \ne 0$ when $|z| = 1$ a single stationary solution of (12) exists and it is given by:

$$X_t = \sum_{j=-\infty}^{\infty} \psi_j \nabla^{-d} Z_{t-j} \qquad (19)$$

where $\Psi(z) = \sum_{j=-\infty}^{\infty} \psi_j z^j = \Theta(z)/\Phi(z)$ (20)

RESCALED-RANGE ANALYSIS (HURST METHOD)

The method of rescaled-range is the oldest approach for assessment of the Hurst's exponent in case of self-similar processes and it should be noted for its very good numerical features. (H.E.Hurst, 1951), (R.Weron, 2002)

Let us define the statistic

$$\frac{R}{S}(n) = \frac{1}{S(n)}\left[\max_{0\le t\le n}\left(Y(t) - \frac{t}{n}Y(n)\right) - \min_{0\le t\le n}\left(Y(t) - \frac{t}{n}Y(n)\right)\right] \tag{21}$$

,where

$Y(t) = \sum_{j=1}^{t} X_t$ are partial sums of the process $\{X_t\}$, and $S^2(n)$ is the empirical dispersion.

In the work (Avram, 1986) is proven that the asymptotical behavior of the R/S_n statistics is like $n^{d+1/2}$ when $n \to \infty$. This result allows for direct assessment of *d* by a linear regression of the logarithms of R/S_n and *n*.

WHITTLE METHOD

Let us take a look at a fractional ARIMA (p, d, q) process $\{X_t\}$, defined by the equation

$$\Phi(B)X_n = \Theta(B)\nabla^{-d}Z_t \quad , \tag{22}$$

where the innovations Z_t are i.i.d. random variables with a zero mathematical expectation. The task is to assess (p+q+1)-dimensional array of parameters $\beta = (\phi_1, \phi_2, \ldots, \phi_p, \theta_1, \theta_2, \ldots, \theta_q, d)$, where $\phi_1, \ldots, \phi_p$ and $\theta_1, \ldots, \theta_q$ are coefficients respectively of the polynomials $\Phi(B), \Theta(B)$ that respect the conditions for the existence and invertibility of FARIMA (p, d, q) process, and the coefficient for fractional differentiation d is supposed to be in the interval $(0,1-1/\alpha)$.

We introduce normalized periodogram

$$\widetilde{I}_n(\omega) = \left(\sum_{t=1}^{n} X_t^2\right)^{-1}\left|\sum_{t=1}^{n} X_t e^{-i\omega t}\right|^2, \quad -\pi \le \omega \le \pi \tag{23}$$

and a power transfer function that depends on the vector β:

$$g(\omega, \beta) = \left|\frac{\Theta(e^{-i\omega}, \beta)}{\Phi(e^{-i\omega}, \beta)(1 - e^{-i\omega})^{d(\beta)}}\right|^2, \quad -\pi \le \omega \le \pi \tag{24}$$

The assessment $\hat{\beta}$ of the parameter vector β is found by finding the minimum of the function

$$\sigma_n^2(\beta) = \int_{-\pi}^{\pi} \frac{\widetilde{I}_n(\omega)}{g(\omega,\beta)} d\omega \tag{25}$$

where the integral is replaced by a sum on Furier' frequencies $\omega_j = 2\pi j / n \in (-\pi, \pi)$:

$$\hat{\sigma}_n^2 = \frac{2\pi}{n} \sum_j \frac{\widetilde{I}_n(\omega_j)}{g(\omega_j, \beta)} \tag{26}$$

WAVELETS METHOD

The wavelet transform appears to be a main tool for studying the scaling properties of a self-similar process (Veitch, D.; Abry, P., Apr 1999). In the current paper the author has applied an estimator proposed by Flandrin (Flandrin, 1992), which estimates Hurst parameter H using the slope of the log-log plot of the detail variance versus the level. A more recent extension can be found in Abry et al. (Abry, P.; P. Flandrin, M.S. Taqqu, D. Veitch , 2003)

MONTE CARLO SIMULATIONS UNDER LONG MEMORY IN RETURNS

The Monte Carlo approach is performed according to the following Algorithm 2:

1. Specify asymmetric stochastic AR (1) / GJR (1, 1) process that models well the dynamics of the capital markets under investigation. Additionally, the standardized residuals of each index are modeled as a standardized Student's *t* distribution to compensate for the fat tails often associated with equity returns.
2. Having filtered the model residuals from each return series, standardize the residuals by the corresponding conditional standard deviation. These standardized residuals represent the underlying zero-mean, unit-variance, i.i.d. series upon which the Extreme Value Theory (EVT) estimation of the tails and sample cumulative distribution function (CDF) of each asset using a generalized Pareto distribution (GPD) estimate for the upper and lower tails.
3. Then, by extrapolating into the generalized Pareto tails and interpolating into the smoothed interior, transform the uniform random variables to standardized residuals via the inversion of the semi-parametric CDF of each index. This produces simulated standardized residuals consistent with those obtained from the AR (1) / GJR (1, 1) filtering process above.
4. 1000 independent random trials of dependent standardized index residuals over a one trading day horizon are simulated. After that long range dependence, estimated on time horizon used for ARMA/GJR estimation is introduced to the simulated residuals.
5. Using the simulated standardized residuals as the i.i.d. input noise process, reintroduce the autocorrelation and heteroscedasticity observed in the original index returns.

6. Having simulated the returns of each index, compute the VaR at 1% confidence level over the one trading day risk horizon.
7. Repeat steps 1 to 6 many times to form the distribution of the over the 1950 trading days horizon with 1024 trading days window.

EVALUATION FRAMEWORK

We have to analyze the models that we have employed to calculate the VaR forecasts in order to assess how much the forecasts reflect the actual market risk in the case of the capital markets of the Balkans. Thus we assess their statistical accuracy through a series of standard tests and in particular: the Kupiec's Test - *likelihood ratio unconditional coverage* and the Christoffersen's Test - *likelihood ratio independence coverage and likelihood ratio conditional coverage*. At this stage we accept as adequate only those models for VaR estimates of the risk for which each of the standard tests gives a positive assessment of adequacy.

For this purpose we define the following error function:

$$F_t = \begin{pmatrix} = 1 & \mathrm{VaR}_t < R_t \\ = 0 & \mathrm{VaR}_t \geq R_t \end{pmatrix} \qquad (27)$$

KUPIEC'S TEST

This test was proposed in 1995 and is the most renowned method to test the adequacy of the models used for VaR forecasting. It is also known as POF- test (proportion of failures), and it provides an assessment of whether the number of exceptions is consistent with (corresponding to) the confidence interval.

In his publication (Kupiec, 1995) proves that the number of these exceptions $\boldsymbol{S} = \sum_1^{\mathrm{T}} \boldsymbol{F_{t+1}}$ is with binomial distribution *B (T, α)*, where *T* is the number of observations. Thus we can appoint a model for VaR(α) forecasts of the risk as adequate if it has an empirical evaluation $\widehat{\boldsymbol{\alpha}} = \sum_1^{\boldsymbol{T}} \boldsymbol{F_{t+1}} / \mathrm{T} * \mathbf{100}[\%]$, which is equal to the value *α* that is set during the definition of the VaR(α) model. The null hypothesis that we test is defined as H_0: $\alpha = \widehat{\boldsymbol{\alpha}} = \frac{\boldsymbol{S}}{\boldsymbol{T}}$ against the alternative H_1 :$\alpha = \widehat{\boldsymbol{\alpha}} \neq \frac{\boldsymbol{S}}{\boldsymbol{T}}$ with a test statistic:

$$LR_{POF} = 2\left[\log\left(\left(\frac{S}{T}\right)^{S}\left(1-\frac{S}{T}\right)^{T-S}\right) - \log\left(\alpha^{S}(1-\alpha)^{T-S}\right)\right] \qquad (28)$$

LR_{POF} is with χ^2 distribution with one degree of freedom.

If the value of the LR_{POF} - statistics exceeds the critical value of the χ^2 distribution, the null hypothesis cannot be accepted and the model will be evaluated as incorrect. For the BACKTESTING process is used 95% of the χ^2 distribution as a critical value for all tests of credibility.

The Kupiec test can accept the models for forecasting VaR values where the number of exceptions is consistent (corresponds to) the confidence interval, but at the same time they produce clustered underestimated forecasts. Then in the periods that follow the undervalued VaR estimate, the probability to have once more underestimated VaR estimate greatly exceeds the confidence interval, and from this perspective the model for forecasting of VaR estimates does not accurately reflect the actual market risk. This problem is discussed by (Christoffersen, 1998) in his paper from 1998.

CHRISTOFFERSEN'S TEST

Christoffersen (Christoffersen, 1998) uses the same log likelihood testing framework as Kupiec, but extends the test to include also a separate statistic for independence of exceptions. The likelihood ratio test statistic is:

$$LR_{CC} = LR_{POF} + LR_{ind} \xrightarrow[T \to \infty]{d} \chi^2(2) \tag{29}$$

LR_{CC} is with χ^2 distribution with two degrees of freedom.

If the value of the LR_{CC}-statistics exceeds the critical value of the χ^2 distribution with two degrees of freedom, the null hypothesis cannot be accepted and the model will be assessed as incorrect. For the BACKTESTING process is used 95% of the χ^2 distribution with two degrees of freedom as a critical value for all tests of credibility.

DATA DESCRIPTION

All analysis undertaken in this paper is based on four Balkans stock markets (Turkey, Croatia, Romania and Bulgaria) in the period $Q_1$2002 - $Q_1$2014. The results that were obtained concern the indices that were surveyed - BET, CROBEX, ISE100 and SOFIX, measured as the daily logarithmic stock returns.

EMPIRICAL RESULTS

In this section, we analyze the accuracy of the VaR estimated maximum probable loss earned on the next trading day obtained with HS, Normal distribution, Student t distribution, Monte Carlo simulations (Algorithm 1) and also with the Monte Carlo simulations under long memory in returns (Algorithm 2). Table 1 shows the results.

Table 1. Accuracy of the VaR estimated maximum probable loss earned on the next trading day

BET					
Method	***failure rate***	***Uncond. Test***	***Ho***	***Cond. Test***	***Ho***
HS	1,12%	0,270893309	accept	10,49469247	reject
Normal distribution	1,93%	13,5773632	reject	29,82304788	reject
Student t distribution	1,22%	0,904134997	accept	10,09184226	reject
Monte Carlo (Algorithm 2) (R/S)	1,07%	0,09024833	accept	0,09024833	accept
Monte Carlo (Algorithm 2) (Whittle Method)	1,12%	0,270893309	accept	0,270893309	accept
Monte Carlo (Algorithm 2) (Wavelets Method)	1,02%	0,00590575	accept	0,00590575	accept
Monte Carlo (Algorithm 1)	1,12%	0,270893309	accept	1,618974287	accept

CROBEX					
method	***failure rate***	***Uncond. Test***	***Ho***	***Cond. Test***	***Ho***
HS	1,42%	3,158417912	accept	20,87731986	reject
Normal distribution	2,59%	35,05779863	reject	66,38154397	reject
Student t distribution	1,58%	5,621066727	reject	26,82232735	reject
Monte Carlo (Algorithm 2) (R/S)	1,02%	0,00590575	accept	0,00590575	accept
Monte Carlo (Algorithm 2) (Whittle Method)	1,02%	0,00590575	accept	0,00590575	accept
Monte Carlo (Algorithm 2) (Wavelets Method)	0,92%	0,145698538	accept	0,145698538	accept
Monte Carlo (Algorithm 1)	1,48%	3,910055681	reject	4,486576192	accept
ISE 100					
method	***failure rate***	***Uncond. Test***	***Ho***	***Cond. Test***	***Ho***
HS	1,27%	1,349149808	accept	10,05885938	reject
Normal distribution	2,39%	27,63263235	reject	35,4870631	reject
Student t distribution	1,68%	7,594455981	reject	13,21743452	reject
Monte Carlo (Algorithm 2) (R/S)	1,07%	0,09024833	accept	1,586461396	accept
Monte Carlo (Algorithm 2) (Whittle Method)	1,12%	0,270893309	accept	1,618974287	accept
Monte Carlo (Algorithm 2) (Wavelets Method)	1,17%	0,543507515	accept	5,375646733	accept
Monte Carlo (Algorithm 1)	1,37%	2,479255898	accept	6,165012091	reject
SOFIX					
method	***failure rate***	***Uncond. Test***	***Ho***	***Cond. Test***	***Ho***
HS	1,27%	1,349149808	accept	21,44664856	reject
Normal distribution	2,49%	31,26147356	reject	69,87192193	reject
Student t distribution	1,42%	3,158417912	accept	26,94474592	reject
Monte Carlo (Algorithm 2) (R/S)	0,61%	3,501800005	accept	3,501800005	accept
Monte Carlo (Algorithm 2) (Whittle Method)	0,86%	0,380959479	accept	0,380959479	accept
Monte Carlo (Algorithm 2) (Wavelets Method)	0,66%	2,588207597	accept	2,588207597	accept
Monte Carlo (Algorithm 1)	1,17%	0,543507515	accept	0,543507515	accept

CONCLUSIONS

This paper presents an empirical analysis of the value-at-risk in the financial environment of the regulated financial markets on the Balkans (Turkey, Croatia, Romania and Bulgaria).The results obtained for the considered stock exchange indices BET, CROBEX, ISE100 and SOFIX indicate presence of long-term dependencies in the logarithmic returns and variance, and respectively the returns are featured by the so called "fat tails" and the assumption of a normal distribution of returns is inappropriate.

At all indices under survey the $VaR_{(1\%)}$ estimates which were calculated for the period $Q_1$2002-$Q_1$2014 through historical simulation and under the assumption of normal and Student t distribution of the returns underestimate the actual market risk and respectively the results we have received evaluate the models as inaccurate. When estimating a Monte Carlo simulation which includes a model of the conditional heteroscedasticity without any long-term dependency the tests for adequacy of the model do not give unambiguous results. The models of conditional heteroscedasticity proposed by the authors consider the long-term dependency computed in three different classical methods (Rescaled-Range Analysis (Hurst Method), Whittle Method and Wavelets Method). They pass successfully through the test of adequacy and generally provide more accurate $VaR_{(1\%)}$ forecasts.

For the purposes of this analysis the following statistical tests are used: Kupiec's test-likelihood ratio unconditional coverage test and Christoffersen's test - likelihood ratio independence coverage and likelihood ratio conditional coverage test.

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